Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Belt Drives

There are two types of drives rigid and flexible. Gear drives are called as rigid or non flexible drives.
Belt, chain and rope drives are called as flexible drives. In flexible drives there is a intermediate link such as belt, chain or rope in-between the driving and driven shafts.

Advantages of flexible drives over rigid drives are as follows:

i) Flexible drives transmit power over a comparatively long distance due to an intermediate link between driving and driven shafts.
ii) Since the intermediate link is long and flexible, it absorbs shock loads and damps vibrations.
iii) Flexible drives provide considerable flexibility in the location of the driving and driven shafts.
iv) Flexible drives are cheap compared to rigid drives . Their initial and maintenance costs are low.

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

The disadvantages of flexible drives are as follows:

i) They occupy more space.
ii) The velocity ratio is relatively small.

Depending on the shape of the cross section, belts are classified as Flat belts and V-belts.

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

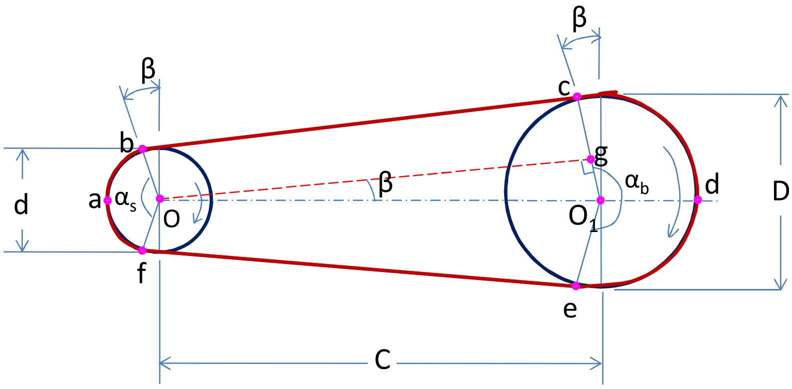
www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Geometrical Relationships :

There are two types of constructions

i) Open belt drive

ii) Crossed belt drive



α_s = wrap angle for small pulley (degrees)
α_b = wrap angle for big pulley (degrees)
D = diameter of big pulley (mm)
d = diameter of small pully(mm), **C** = centre distance (mm)

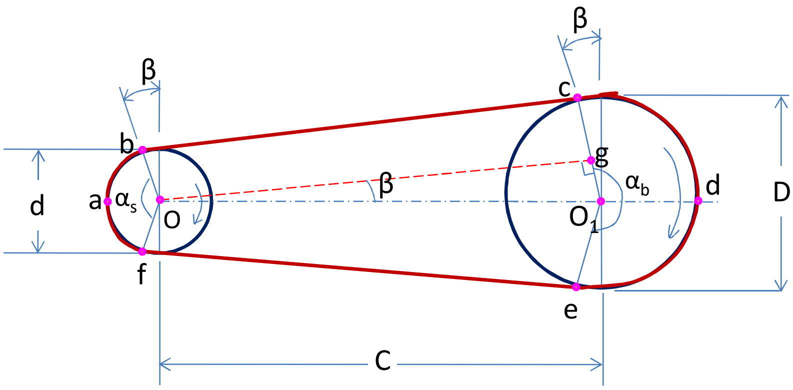
Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Geometrical Relationships :

There are two types of constructions

i) Open belt drive



Length of the belt

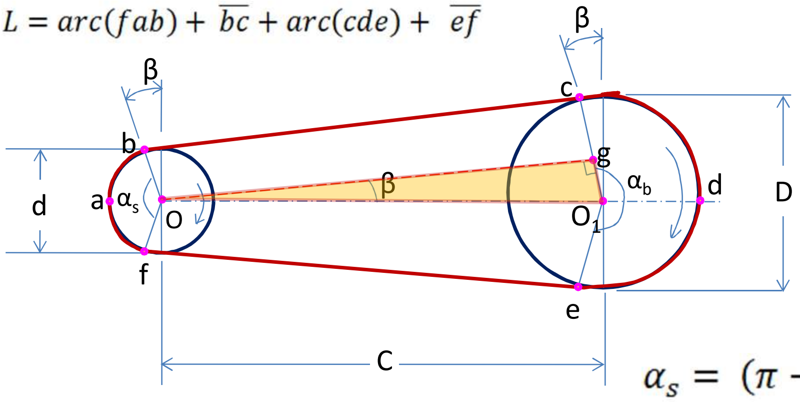
$$L = arc(fab) + \overline{bc} + arc(cde) + \overline{ef}$$

Geometrical Relationships :

There are two types of constructions

i) Open belt drive

$$L = arc(fab) + \overline{bc} + arc(cde) + \overline{ef}$$



$$\alpha_s = (\pi - 2\beta)$$

$$arc(fab) = \frac{d}{2} \times \alpha_s$$

$$Sin\beta = \beta = \frac{(D-d)}{2C}$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

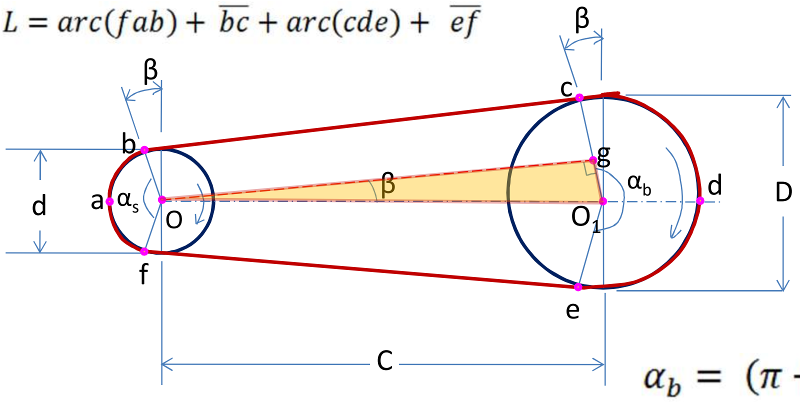
www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Geometrical Relationships :

There are two types of constructions

i) Open belt drive

$$L = arc(fab) + \overline{bc} + arc(cde) + \overline{ef}$$



$$\alpha_b = (\pi + 2\beta)$$

$$arc(cde) = \frac{D}{2} \times \alpha_b$$

$$Sin\beta = \beta = \frac{(D-d)}{2C}$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

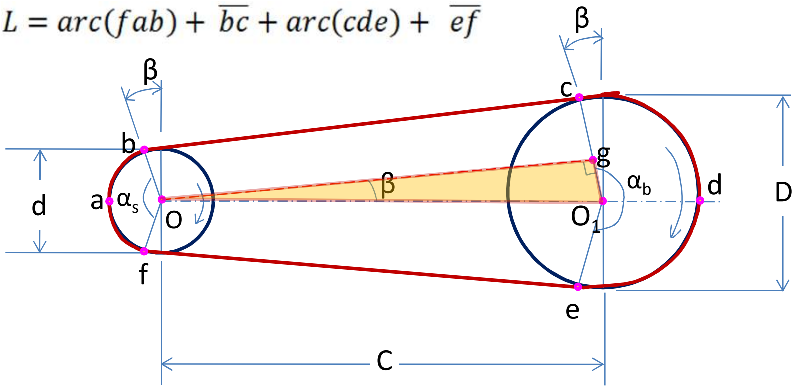
www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Geometrical Relationships :

There are two types of constructions

i) Open belt drive

$$L = arc(fab) + \overline{bc} + arc(cde) + \overline{ef}$$



$$\overline{bc} = \overline{ef} = C\cos\beta$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

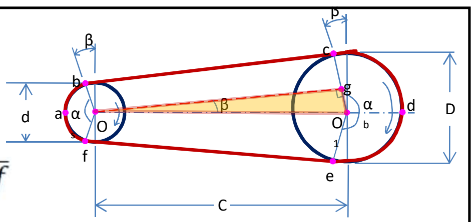
www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Geometrical Relationships :

There are two types of constructions

i) Open belt drive

$$L = arc(fab) + \overline{bc} + arc(cde) + \overline{ef}$$



$$L = \frac{d}{2}(\pi - 2\beta) + C.Cos\beta + \frac{D}{2}(\pi + 2\beta) + C.Cos\beta$$

$$L = \frac{\pi}{2}(D + d) + 2C.Cos\beta + \beta(D - d)$$

Trigonometric Formula

$$\boxed{Cos\beta = 1 - 2Sin^2\left(\frac{\beta}{2}\right)}$$

$$Cos\beta = 1 - 2\left(\frac{\beta^2}{4}\right)$$

$$Sin\beta = \beta = \frac{(D-d)}{2C}$$

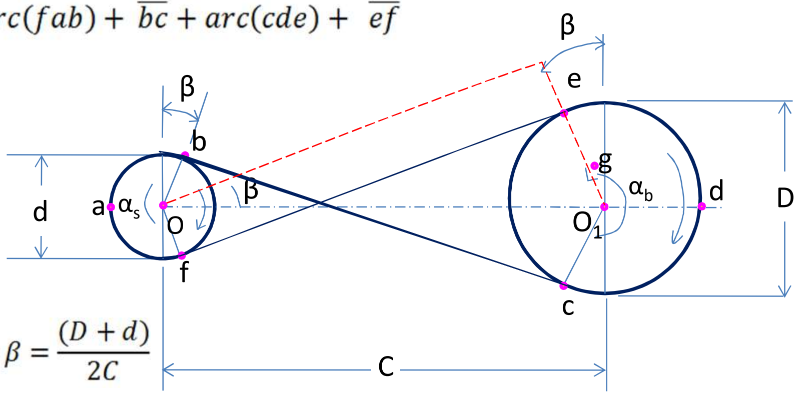
$$L = \frac{\pi}{2}(D + d) + \left[2C.\left(1 - \frac{(D-d)^2}{8C^2}\right)\right] + \left[\frac{(D-d)}{2C}(D - d)\right]$$

$$L = \frac{\pi}{2}(D + d) + 2C + \frac{(D-d)^2}{4C}$$

Geometrical Relationships :

ii) Cross belt drive

$$L = arc(fab) + \overline{bc} + arc(cde) + \overline{ef}$$



$$Sin\beta = \beta = \frac{(D+d)}{2C}$$

$$\alpha_s = \alpha_b = (\pi + 2\beta)$$

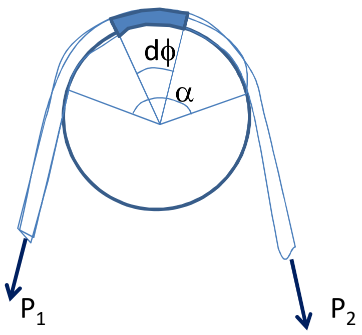
$$L = \frac{\pi}{2}(D+d) + 2C + \frac{(D+d)^2}{4C}$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Analysis of Belt Tensions

The forces acting on the element of flat belt are shown in figure.
The following notations are used in derivations:



P_1 = Belt tension on tight side (N)
P_2 = Belt tension on loose side (N)
m = mass of one metre length of belt (kg/m))
v = belt velocity (m/s)
μ = coefficient of friction
α = angle of wrap (radians)

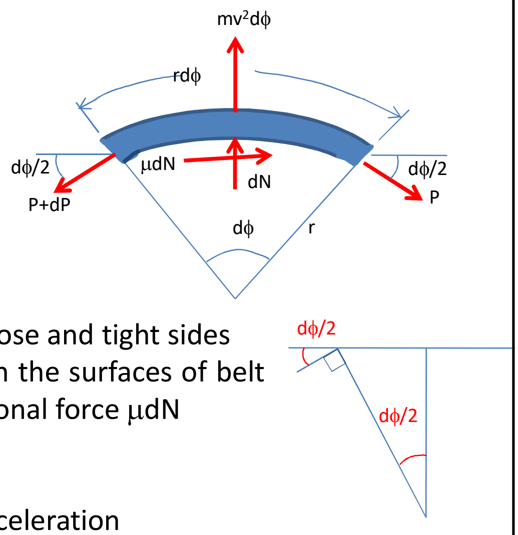
Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Dr. M.Raja Roy
B.Tech,M.E,Ph.D
www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Analysis of Belt Tensions

An element of belt subtended by an angle dϕ is in equilibrium under the action of the following forces :



i) Tensions (P) and (P+dP) on loose and tight sides
ii) The normal reaction between the surfaces of belt and pulley (dN) and the frictional force μdN
iii) Centrifugal force

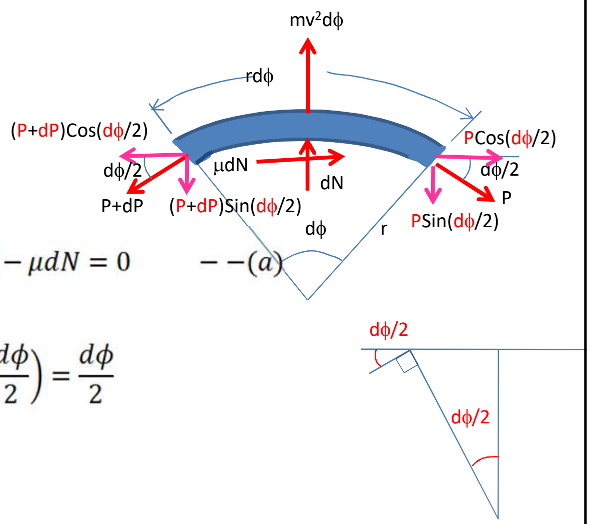
Centrifugal force = mass X acceleration

$$= (m.rd\phi) \times \left(\frac{v^2}{r}\right)$$
$$= (mv^2d\phi)$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D
www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Analysis of Belt Tensions

Consider the equilibrium of forces in X direction :



$$(P+dP)cos\left(\frac{d\emptyset}{2}\right) - Pcos\left(\frac{d\emptyset}{2}\right) - \mu dN = 0 \qquad --(a)$$

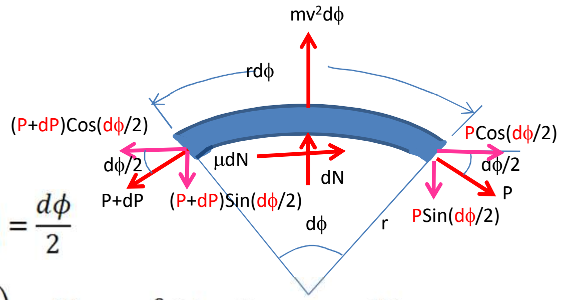
$$cos\left(\frac{d\phi}{2}\right) = 1 \;\; and \;\; sin\left(\frac{d\phi}{2}\right) = \frac{d\phi}{2}$$

$$P + dP - P - \mu dN = 0$$

$$dN = \frac{dP}{\mu} \qquad --(c)$$

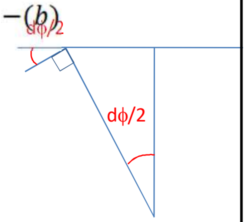
Analysis of Belt Tensions

Consider the equilibrium of forces in Y direction :



$$\cos\left(\frac{d\phi}{2}\right) = 1 \;\; and \;\; \sin\left(\frac{d\phi}{2}\right) = \frac{d\phi}{2}$$

$$(P + dP)\sin\left(\frac{d\emptyset}{2}\right) + P\sin\left(\frac{d\emptyset}{2}\right) - dN - mv^2 d\phi = 0 \qquad --(b)$$



$$P\left(\frac{d\emptyset}{2}\right) + \left(\frac{dPd\emptyset}{2}\right) + P\left(\frac{d\emptyset}{2}\right) - dN - mv^2 d\phi = 0$$

$$Pd\emptyset - dN - mv^2 d\phi = 0$$

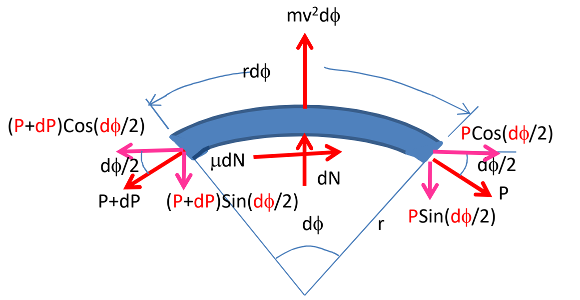
$$dN = Pd\emptyset - mv^2 d\phi$$

$$\frac{dP}{\mu} = (P - mv^2)\, d\phi \qquad\qquad \frac{dP}{(P - mv^2)} = \mu d\phi$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

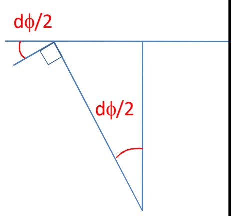
www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Analysis of Belt Tensions



$$\frac{dP}{(P - mv^2)} = \mu d\phi$$

Integrate the above equation



$$\int_{P_2}^{P_1} \frac{dP}{(P - mv^2)} = \mu \int_0^{\alpha} d\phi$$

$$[log(P - mv^2)]_{P_1}^{P_2} = \mu[\phi]_0^{\alpha}$$

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{\mu\alpha}$$

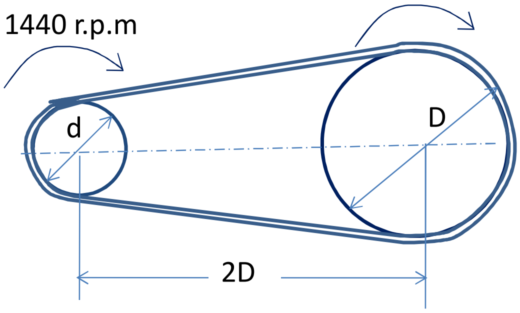
Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

The layout of the leather belt drive transmitting 15KW power is shown in Fig.3. The centre distance between the pulleys is twice the diameter of the big pulley. The belt should operate at a velocity of 20m/s approximately and the stress in the belt should not exceed 2.25N/mm^2. The density of leather is 0.95 gm/cc and the coefficient of friction is 0.35. The thickness of the belt is 5mm. Calculate : i) the diameters of pulleys ii) the length and width of the belt iii) the belt tensions.



Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Given data :

$KW = 15 KW$, $L = 2D$ $\quad v = 20\, m/s$ $\quad \sigma = 2.25\, N/mm^2$

$\rho = 0.95\, gm/cc$ $\quad \mu = 0.35$ $\quad t = 5\, mm$

$n_1 = 1440\, r.p.m$ $\quad n_2 = 480\, r.p.m$

$$V = \frac{\pi d n}{60 \times 10^3} \Rightarrow d = \frac{60 \times 10^3 \times V}{\pi n_1}$$

$$= \frac{60 \times 10^3 \times 20}{\pi \times 1440} = 265.26\, mm$$

$$\sim 270\, mm$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Velocity Ratio i.e $\frac{D}{d} = \frac{n_1}{n_2}$

$$D = \frac{n_1}{n_2} \times d = \frac{1440}{480} \times 270 = 810 \text{ mm}$$

Correct belt velocity $V = \frac{\pi d n_1}{60\times10^3}$

$$= \frac{\pi \times 270 \times 1440}{60\times10^3} = 20.36 \text{ m/s}$$

Length of the Belt

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$= (2\times1620) + \frac{\pi(810+270)}{2} + \frac{(810-270)^2}{4\times1620}$$

$$= 4981.46 \text{ mm}$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 180 - 2\sin^{-1}\left(\frac{810-270}{2\times1620}\right)$$

$$= 160.8^\circ \text{ or } 2.81 \text{ radians}$$

Mass of the belt 'm' per metre length

$$\rho = \frac{m}{V} \Rightarrow m = \rho . V$$

$$= \rho . A . L$$

$$= \rho . A . 1$$

V = volume
A = Area
L = Length
$L = 1$ ∵ per length

$$\rho = 0.95\ gm/cc$$
$$= 0.95 \times \frac{Kg}{1000} / cm^3$$
$$= 0.95 \times 10^{-3} / \left(\frac{1}{100}\right)^3 Kg/m^3$$
$$= 0.95 \times 10^3\ Kg/m$$
$$A = \left(\frac{b}{1000} \times \frac{5}{1000}\right) m^2$$
$$m = 0.95 \times 10^3 \times \frac{5b}{10^6}$$
$$= 0.95 \times 5b \times 10^3 \times 10^{-6}$$
$$= 4.75b \times 10^{-3}\ Kg/m$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

$$mv^2 = 4.75b \times 10^{-3} \times (20.36)^2$$
$$= 1.97b$$
$$e^{\mu\alpha} = e^{0.35 \times 2.81}$$
$$= 2.67$$

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{\mu\alpha}$$
$$\frac{P_1 - 1.97b}{P_2 - 1.97b} = 2.67$$
$$P_1 - 2.67P_2 + 3.29b = 0 \quad \text{—— (i)}$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

$$\sigma = \frac{P_1}{A} \Rightarrow P_1 = \sigma \cdot A$$
$$= 2.25 \times (5b)$$
$$P_1 = 11.25b \quad \text{—— (2)}$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

$$P_1 - P_2 = \frac{1000 \times KW}{v}$$
$$= \frac{1000 \times 15}{20.36}$$
$$P_1 - P_2 = 736.74 \quad \text{—— (3)}$$

$$P_1 = 2.67 P_2 + 3.29 b = 0 \quad \text{—— (1)}$$
$$P_1 = 11.25b \quad \text{—— (2)}$$
$$P_1 - P_2 = 736.74 \quad \text{—— (3)}$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

$P_1 = 1428.98\,N \qquad b = 127.02$ mm

$P_2 = 692\,N$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Centrifugal Tension :

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both the tight as well as the slack sides. The tension caused by centrifugal force is called centrifugal tension.

At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion PQ of the belt subtending an angle dθ at the centre of the pulley, as shown in Figure.

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Consider a small portion PQ of the belt subtending an angle dθ at the centre of the pulley, as shown in Figure.

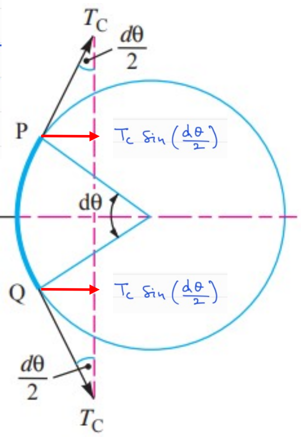
Let

m = Mass of belt per unit length in kg

v = Linear velocity of belt in m/sec

r = Radius of the pully in metres

T_c = Centrifugal Tension in Newtons



Length of the belt PQ $= r.d\theta$

Mass of the belt PQ $= m.r.d\theta$

Centrifugal force on belt PQ

$$= (m.r.d\theta).\frac{v^2}{r}$$

$$F_c = m.d\theta.v^2$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

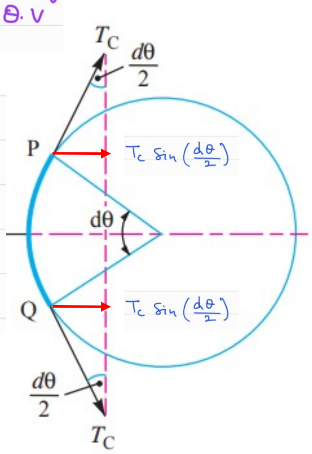
Now resolve the forces horizontally

$$T_c \sin\left(\frac{d\theta}{2}\right) + T_c \sin\left(\frac{d\theta}{2}\right) = m.d\theta.v^2$$

For small values $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$

$$2T_c\left(\frac{d\theta}{2}\right) = m.d\theta.v^2$$

$$T_c = m.v^2$$



Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Effect of Centrifugal Tension on Power

Tension on Tight side $T_{t1} = T_1 + T_c$

Tension on slack side $T_{t2} = T_2 + T_c$

Power Transmitted $P = (T_{t1} - T_{t2}).v$

$$= [(T_1 + T_c) - (T_2 + T_c)].v$$

$$= (T_1 - T_2).v$$

Hence centrifugal Tension is not effecting Power

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Condition for the transmission of Maximum Power

Power Transmitted by belt $P = (T_1 - T_2) . V$ — (1)

T_1 = Tension on tight side (N)
T_2 = Tension on slack side (N)
V = velocity of belt (m/s)

Ratio of driving tensions

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \& \quad T_2 = \frac{T_1}{e^{\mu\theta}}$$ — (2)

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Substituting (2) in (1)

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right) . V$$

$$= T_1\left(1 - \frac{1}{e^{\mu\theta}}\right) . V$$

$$= T_1 . V . C$$ — (3)

where $C = \left(1 - \frac{1}{e^{\mu\theta}}\right)$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

Maximum Tension in belt after taking Centrifugal Tension is

$$T = T_1 + T_c$$

$$T_1 = T - T_c \quad \text{— (4)}$$

Substitute (4) in (3)

$$P = T_1 . v . C$$
$$= (T - T_c) . v . C$$
$$= (T - m . v^2) . v . C$$
$$= (T . v - m v^3) . C$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

For maximum power differentiate Power with velocity and equate to zero

$$\frac{dP}{dv} = 0$$

$$\frac{d}{dv}(T . v - m v^3) . C = 0$$

$$T - 3 m v^2 = 0 \quad \text{— (5)}$$

velocity of Max. Power $v = \sqrt{\dfrac{T}{3 . m}}$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

From (5) $T - 3T_c = 0 \quad \because T_c = mv^2$

$$T_c = \frac{T}{3}$$

For transmitting max power $\frac{1}{3}$ of the max tension is absorbed as centrifugal tension.

Example *A leather belt 9 mm × 250 mm is used to drive a cast iron pulley 900 mm in diameter at 336 r.p.m. If the active arc on the smaller pulley is 120° and the stress in tight side is 2 MPa, find the power capacity of the belt. The density of leather may be taken as 980 kg/m³, and the coefficient of friction of leather on cast iron is 0.35.*

Solution. Given: t = 9 mm = 0.009 m ; b = 250 mm = 0.25 m; d = 900 mm = 0.9 m ; N = 336 r.p.m ; $\theta = 120° = 120 \times \frac{\pi}{180} = 2.1$ rad ; σ = 2 MPa = 2 N/mm² ; ρ = 980 kg/m³ ; μ = 0.35

We know that the velocity of the belt,

$$v = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.9 \times 336}{60} = 15.8 \text{ m/s}$$

and cross-sectional area of the belt,

$$a = b.t = 9 \times 250 = 2250 \text{ mm}^2$$

∴ Maximum or total tension in the tight side of the belt,

$$T = T_{t1} = \sigma.a = 2 \times 2250 = 4500 \text{ N}$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho = 0.25 \times 0.009 \times 1 \times 980 \text{ kg/m}$$
$$= 2.2 \text{ kg/m}$$

$\therefore$ Centrifugal tension,

$$*T_C = m.v^2 = 2.2\,(15.8)^2 = 550 \text{ N}$$

and tension in the tight side of the belt,

$$T_1 = T - T_C = 4500 - 550 = 3950 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.35 \times 2.1 = 0.735$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.735}{2.3} = 0.3196 \quad \text{or} \quad \frac{T_1}{T_2} = 2.085$$

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym

$$T_2 = \frac{T_1}{2.085} = \frac{3950}{2.085} = 1895 \text{ N}$$

We know that the power capacity of the belt,

$$P = (T_1 - T_2)\,v = (3950 - 1895)\,15.8 = 32\,470 \text{ W} = 32.47 \text{ kW}$$ **Ans.**

Dr. M.Raja Roy
B.Tech,M.E,Ph.D

www.mrrtechnical.co.in
www.rajaroy.co.in
www.youtube.com/rajaroym